Using Variational Autoencoders for Dimensionality Reduction of Nonlinear Structural Dynamics

Prepared for:

Scipy 2020

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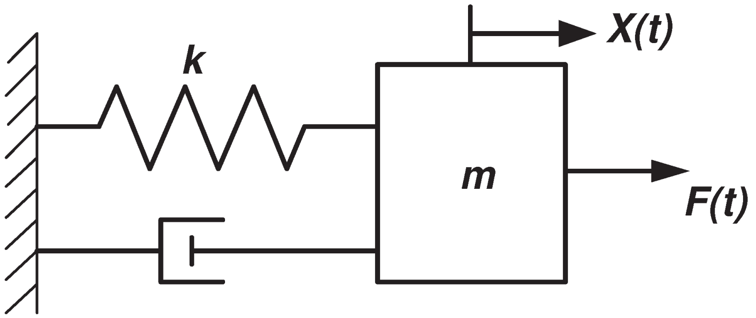
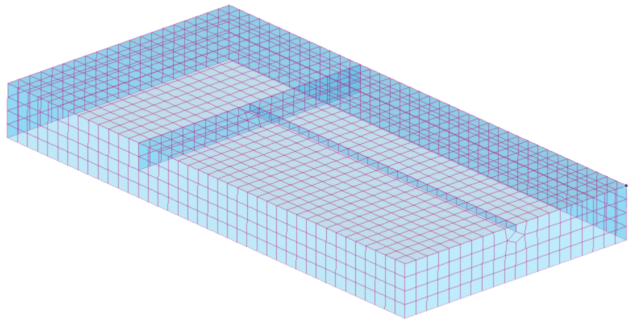
Structural Dynamics Background and Motivation

- Structural dynamics involves studying the vibration of mechanical structures to external excitations
  - Rocket engine acoustic loading, aerodynamic loading, jet engine rotational forces, etc.
- These systems are typically modeled via the Finite Element (FE) method that results in 2nd order ODE that must be integrated in time
  - Systems can be up to **10s of millions of DOF** and integration for long time durations is typically infeasible even with large HPC clusters

$$\mathrm{M}\ddot{\mathrm{x}} + \mathrm{C}\dot{\mathrm{x}} + \mathrm{Kx} = \mathrm{f}_{\mathrm{ext}}$$

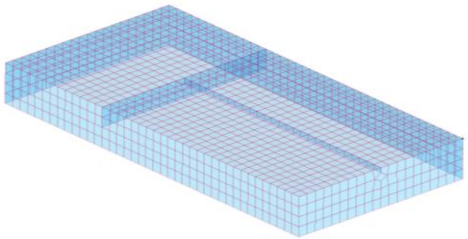


Simple Spring-Damper-Mass System

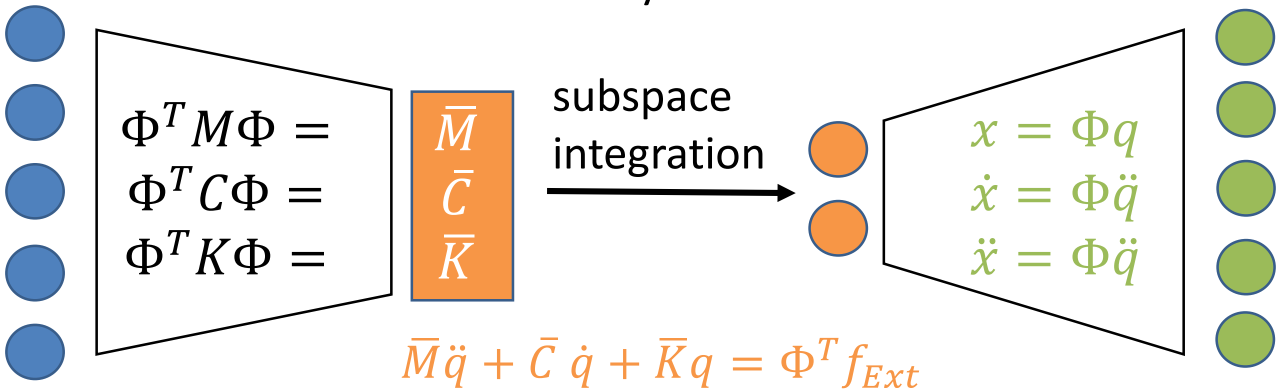


Generic FE Model

Integration of full FE model is too costly for realistic structure so model reduction is performed via modal analysis.



Modal Analysis Workflow



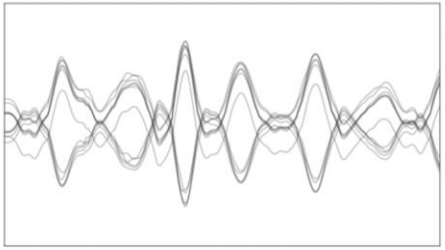
$$\bar{M}\ddot{q} + \bar{C}\dot{q} + \bar{K}q = \Phi^T f_{Ext}$$

Physical Space of Size [n]

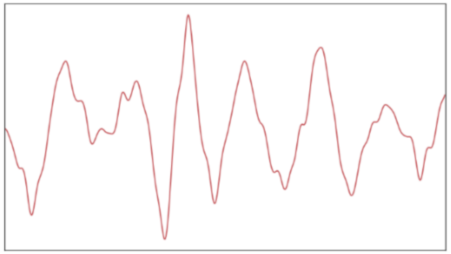
$$\mathrm{M}\ddot{\mathrm{x}} + \mathrm{C}\dot{\mathrm{x}} + \mathrm{K}\mathrm{x} = \mathrm{f}_{\mathrm{ext}}$$

Modal Space of Size [m]

Recovered Data

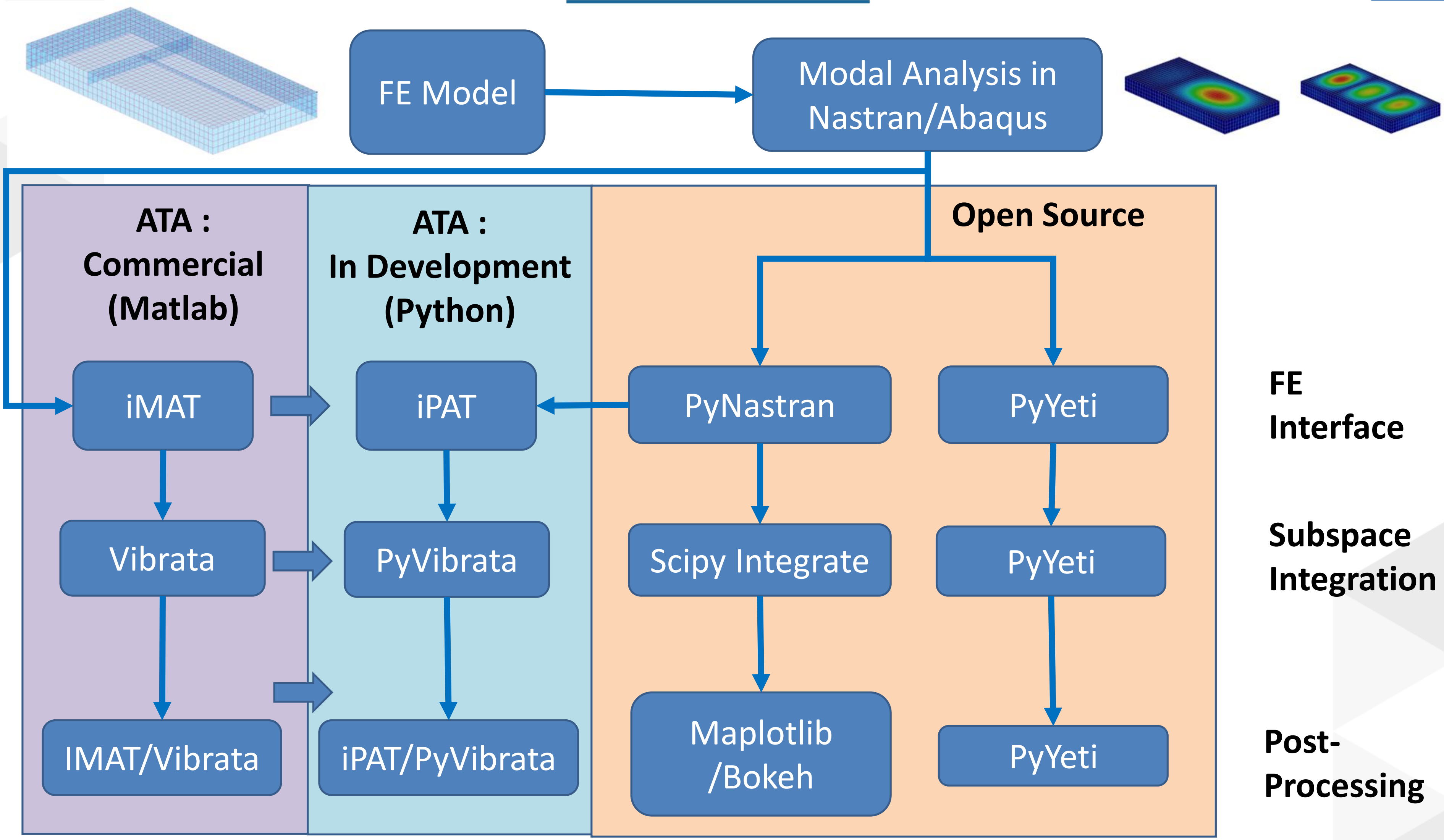


Modal Response



Physical random response

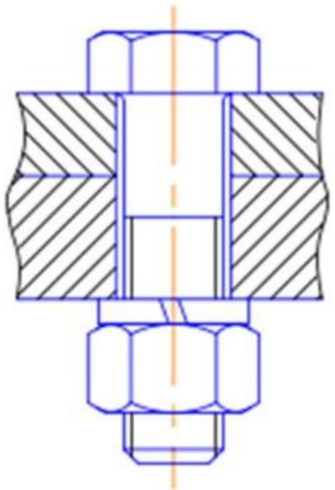
Python is quickly becoming a go to toolset for structural dynamics analysis offering many potential routes.



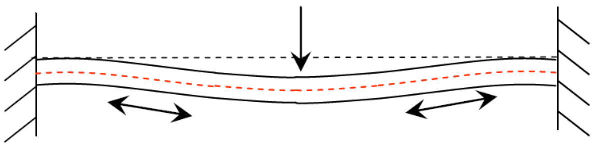
In reality systems are nonlinear and conventional modal analysis reduction techniques become inaccurate.

- Nonlinearities can manifest themselves in a variety of ways within structural dynamics problems
  - Contact
  - Bolted Joints
  - Nonlinear Material (rubbers, plasticity, viscoelasticity)
  - Large deformations (geometric nonlinearitiy)
    - Focus of todays talk
- Linear normal modes (from eigenvalue analysis) are no longer an optimal basis for model reduction

$$M\ddot{x} + C\dot{x} + Kx + f_{nl}(x, \dot{x}) = f_{ext}$$



Geometric nonlinearity is important to account for when structures are expected to have large deformations.

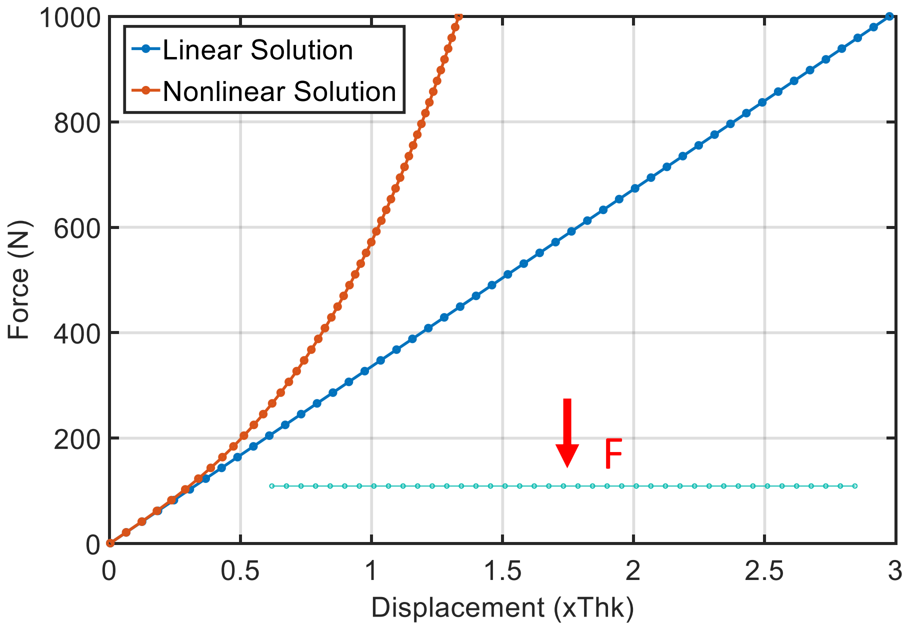


$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}_{ext}$$

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{nl}(\mathbf{x}) = \mathbf{f}_{ext}$$

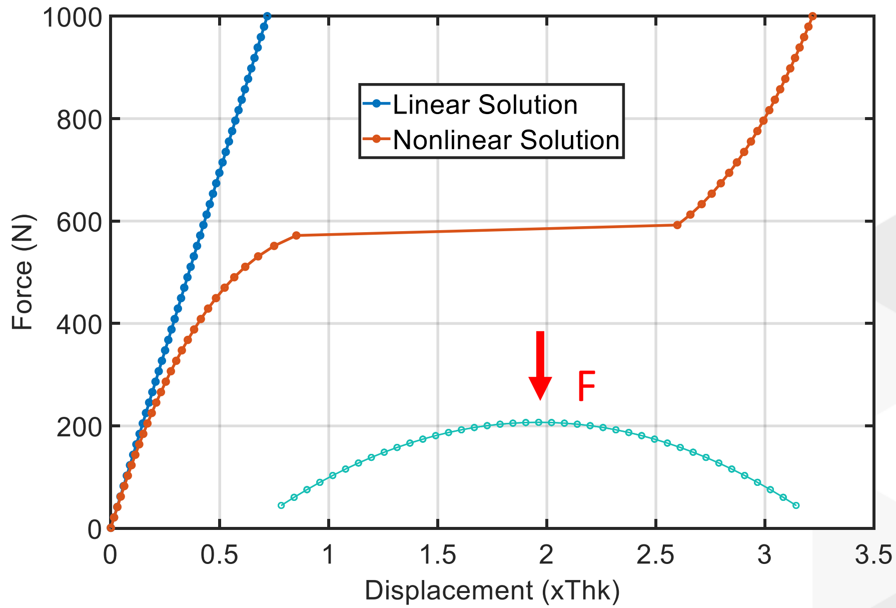
Flat structures

Linear solution procedures overestimate response

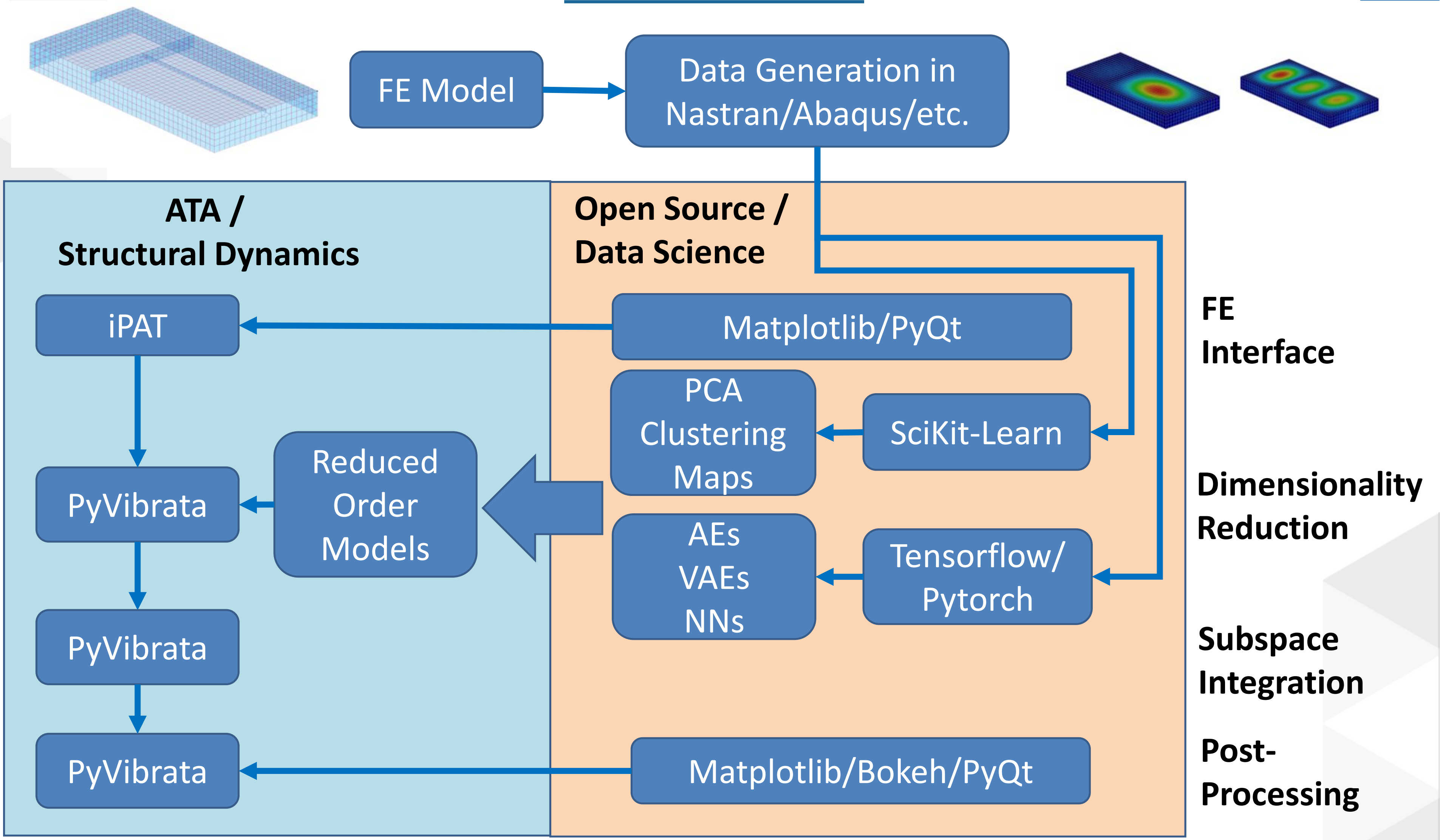


Curved structures

Linear solution procedures may be non-conservative and may not accurately estimate stability boundaries



ATA is developing a framework within Python capable of model reduction via conventional approaches as well as statistically based reduction procedures.



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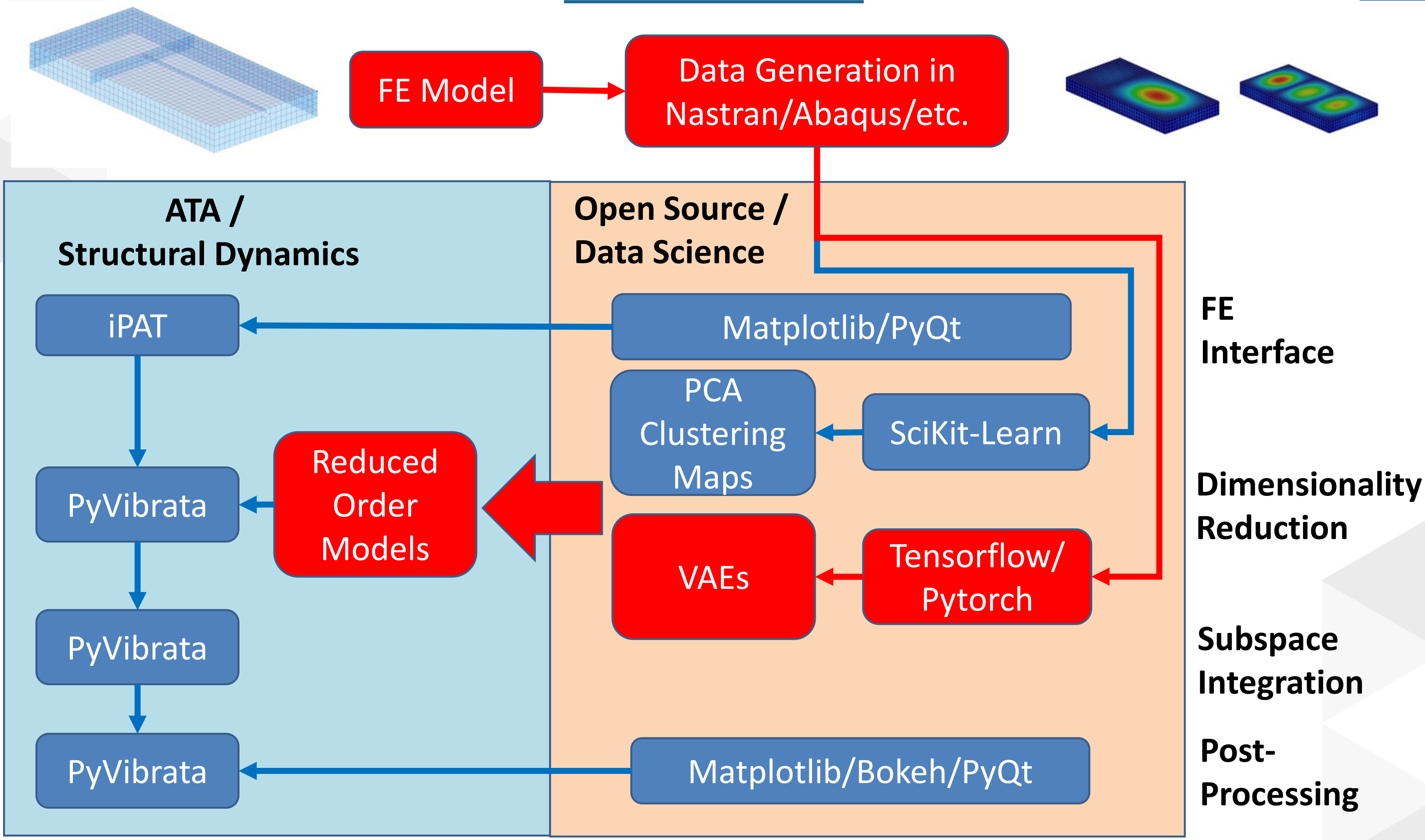


Flat Beam Case Study - Model

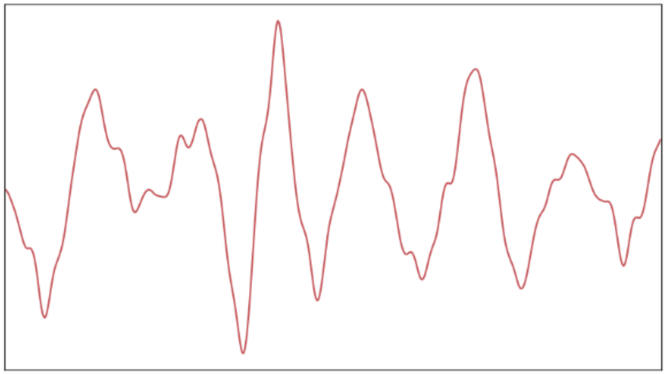
Structure:
- Thin flat beam
- Steel
- 9” x 0.031” x 0.5”
- Used in many previous works [ref][ref][ref]

FE model :
- 40 two-node nonlinear beam elements
- 117 free DOF

Data :
- Generated linear and nonlinear data from 3 different levels of excitation
  - 90 dB, 100 dB, 110 dB

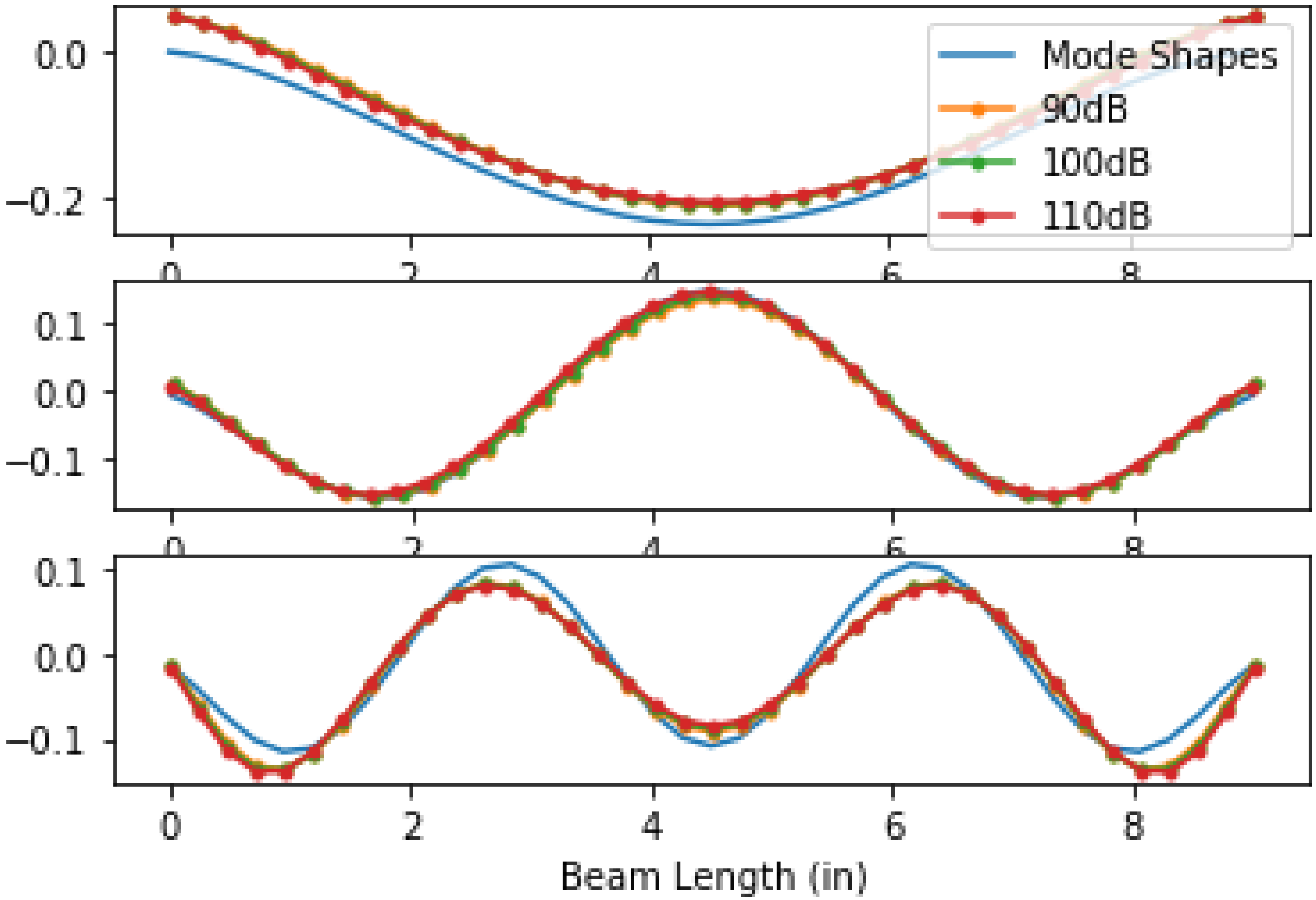
Idealization of the Beam

FEM of the Beam

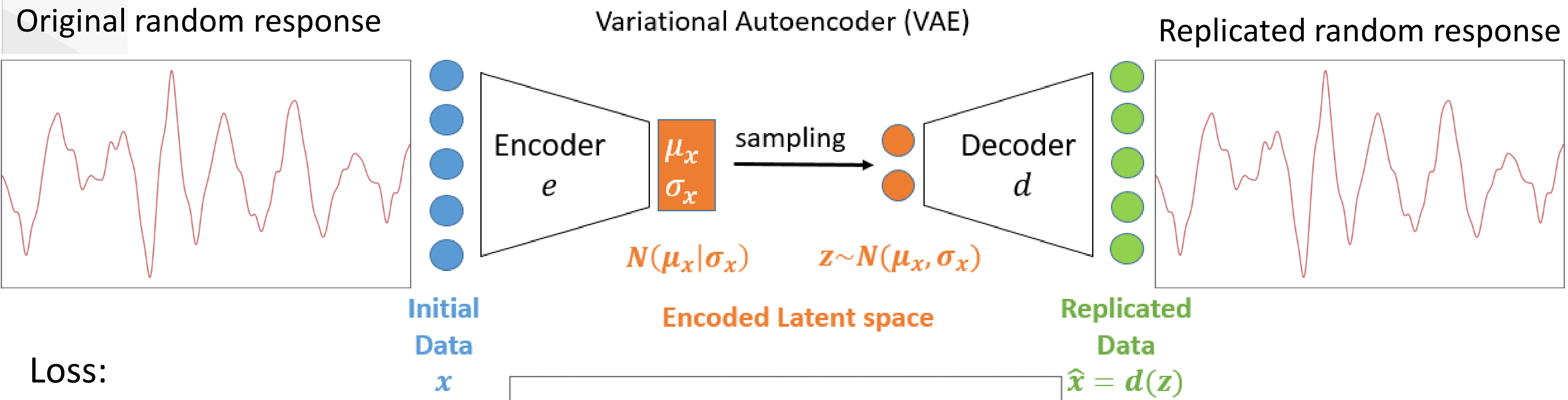


Center Node Response

PCA on training data gives similar shapes as the eigenvectors from modal analysis.

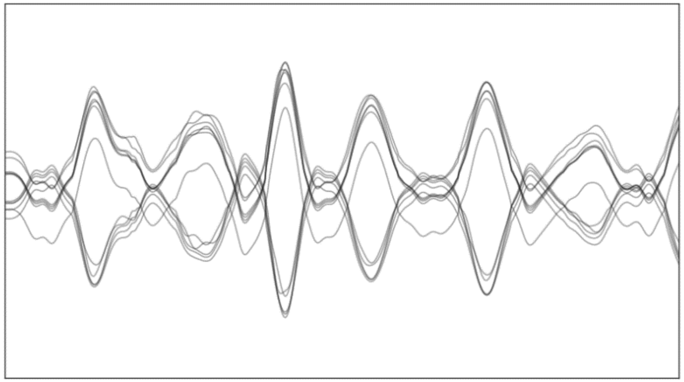


Variational Autoencoder with Orthogonality Constraint

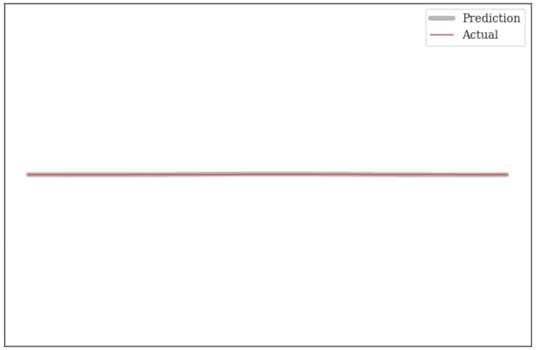


Loss:
Reconstruction +
α*[KL divergence] +
β*[Cosine Similarity]

Low Cosine Similarity indicates orthogonality between mode shapes

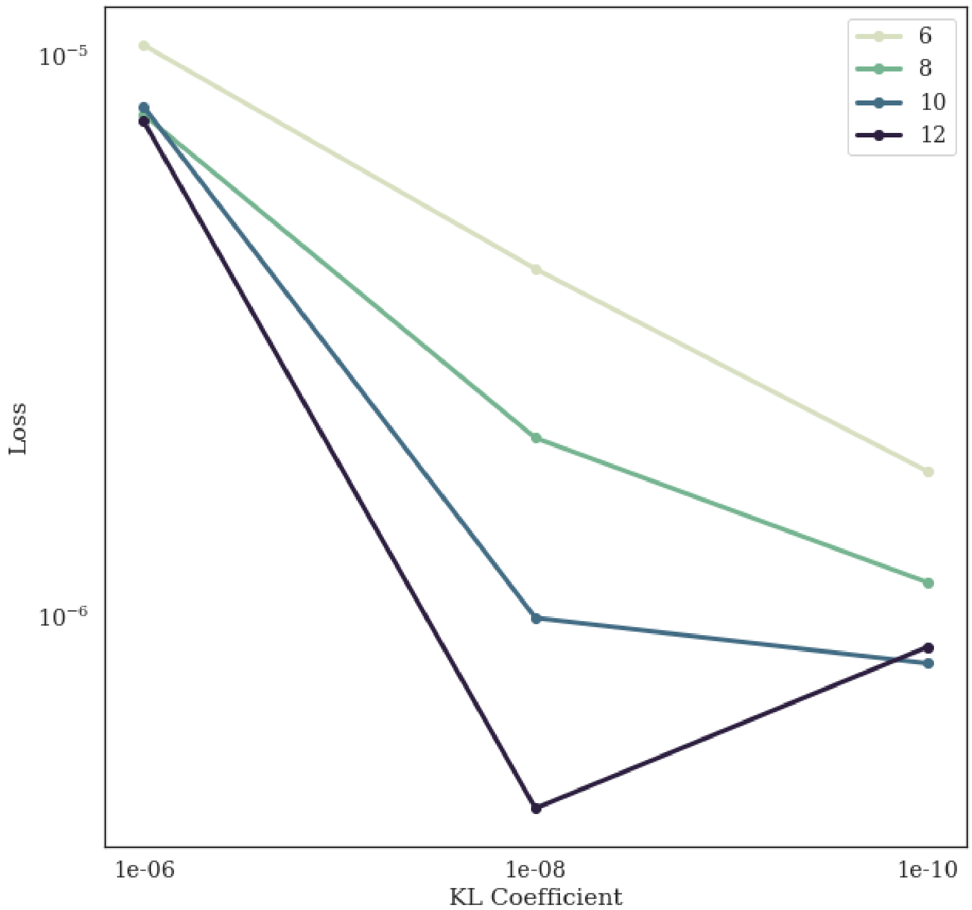


Encoded response



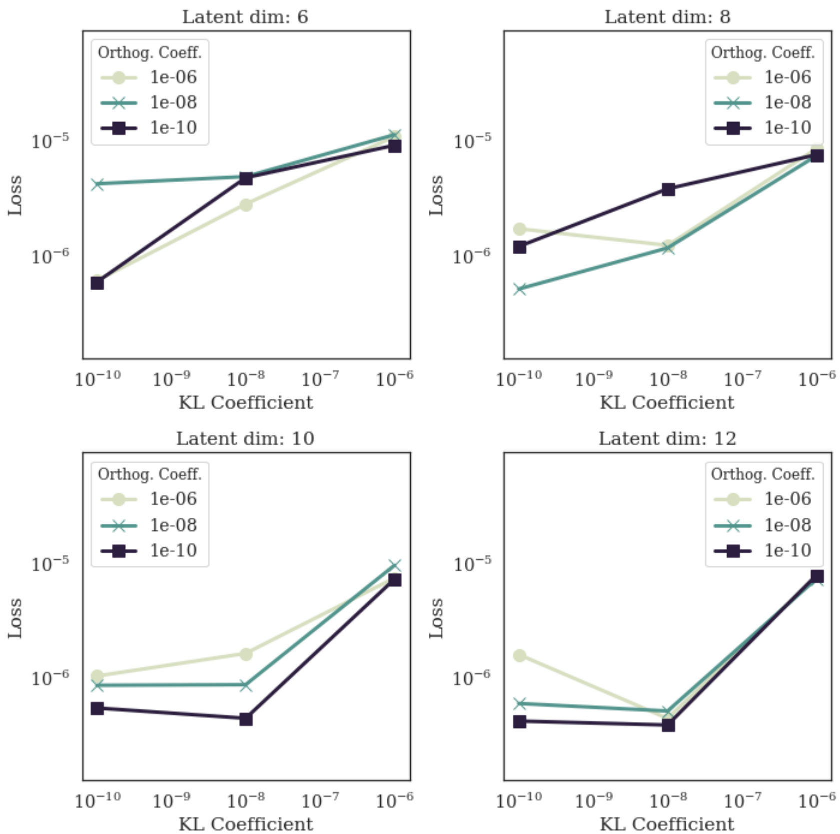
Increasing the size of the latent dimension generally results in a smaller reconstruction loss

- Reconstruction improves with more latent dimensions.
- Likewise, reconstruction improves as the KL divergence constraint is relaxed, except for 12 latent dimensions.

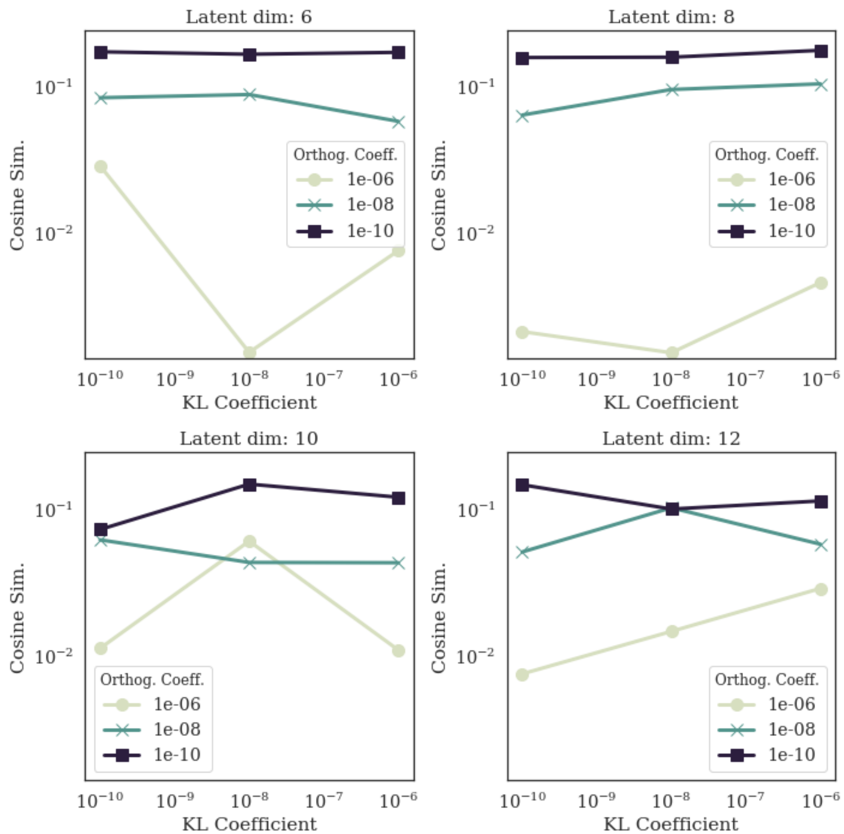


Reconstruction & Cosine Similarity Loss as a function of latent dimension size, orthogonality coefficient, and KL divergence coefficient

Reconstruction Loss

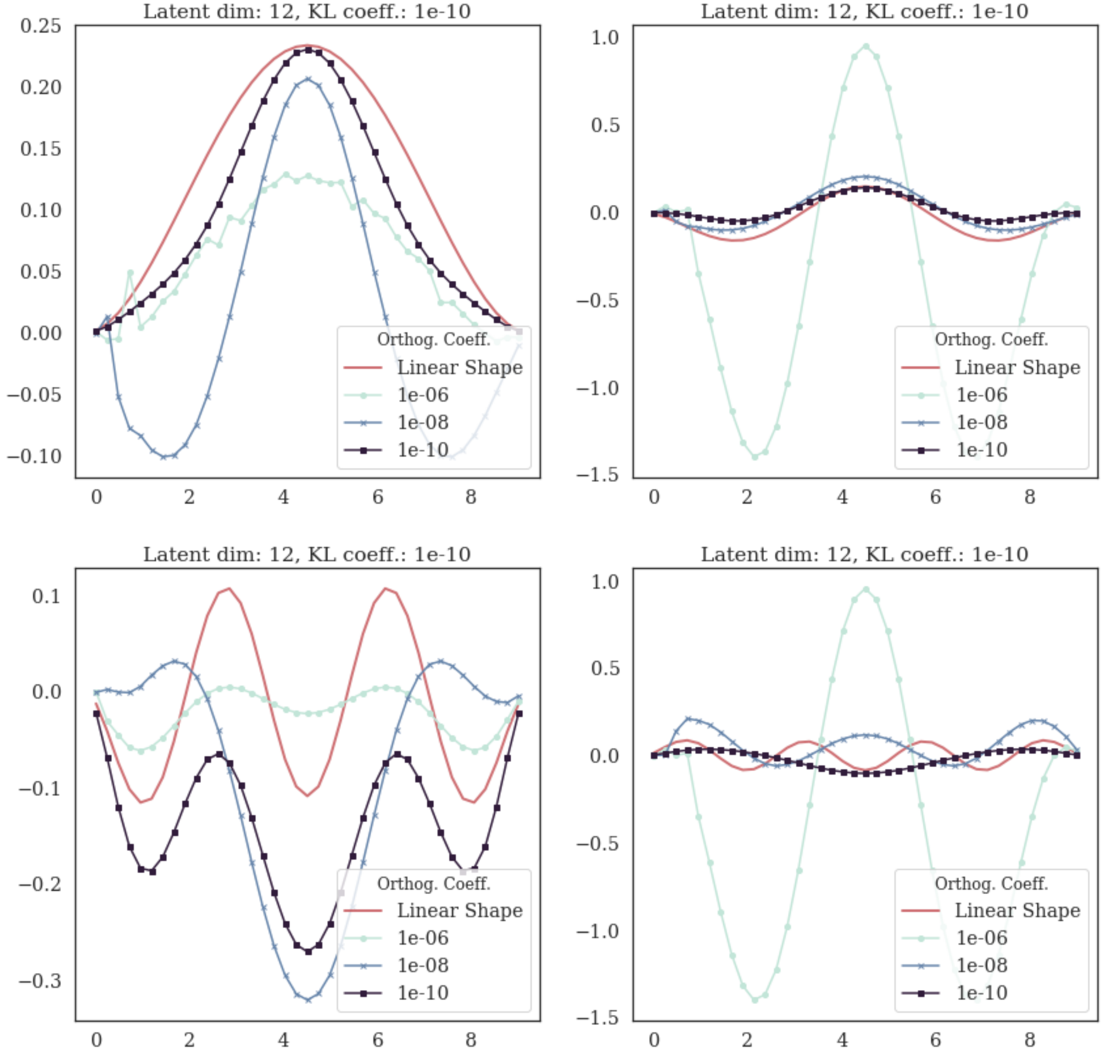


Cosine Similarity Loss



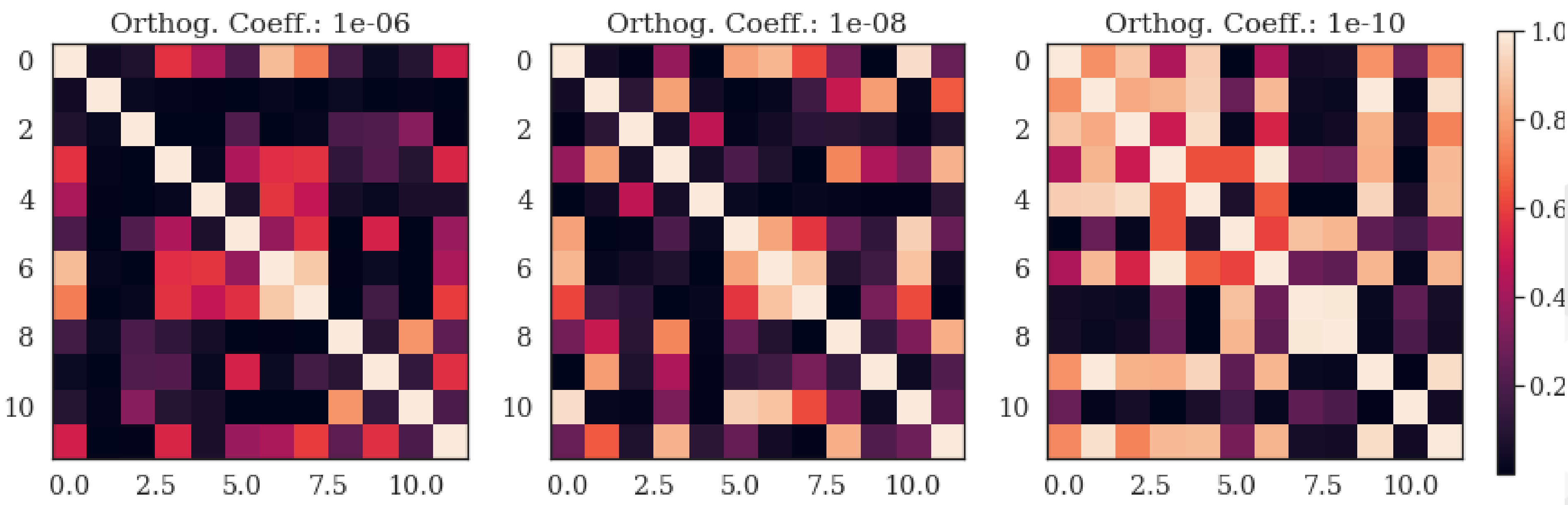
For 10 and 12 latent dimensions, there seems to be an intermediate sweet spot for the KL divergence value.

Shapes extracted by VAE resemble linear mode shapes obtained via eigenanalysis

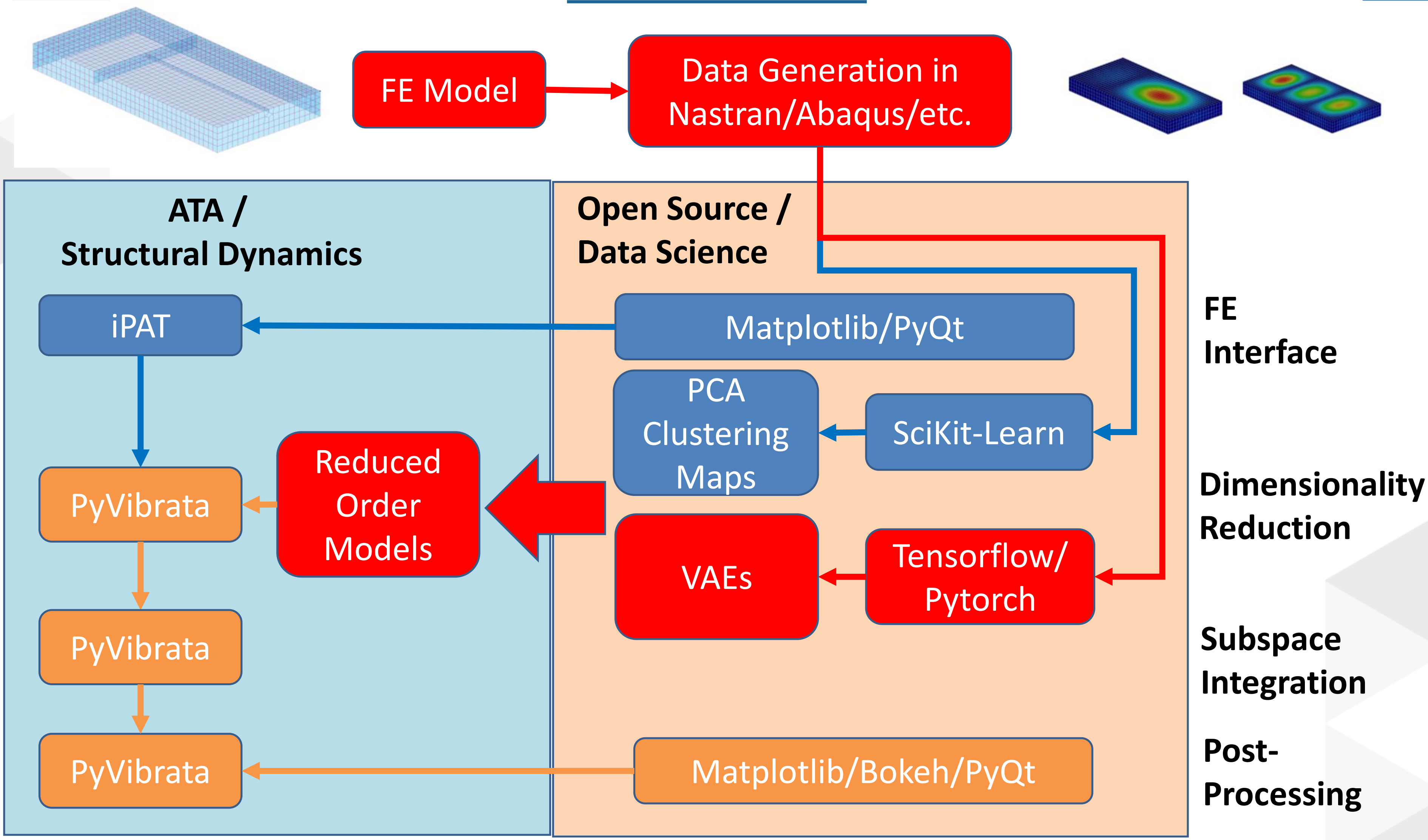


Orthogonality of the output layer weights can be enforced by increasing the weighting on cosine similarity loss function.

- The figures shown below represent the cosine similarity between each output weight in the decoder for a model with 12 latent dimensions
  - This is similar to the mode shapes from modal analysis
- Normal modes from modal analysis would have only diagonal terms of 1.0 and off diagonals of 0.0 representing an orthogonal set



Next steps are to integrate the dimensionality reduction techniques into framework suitable for subspace integration procedures.



Questions?

- All the FE related code is open source at
  - https://cvandamme@bitbucket.org/cvandamme/osfern.git
- All the MHB code is open source at
  - https://cvandamme@bitbucket.org/cvandamme/wiscmhb.git

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